Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Total put in.	*Alcohol put in.*	*Total.*
1	c	1
p	ap	$1+p$
$(1+p)p$	$(1+p)ap$	$(1+p)^2$
$(1+p)^2p$	$(1+p)^2ap$	$(1+p)^3$
$\vdots$	$\vdots$	$\vdots$
$(1+p)^np$	$(1+p)^nap$	$(1+p)^{n+1}$

The sum of the quantities in the second column is $c-a+a(1+p)^{n+1}$, and this divided by $(1+p)^{n+1}$ is the answer.

GEOMETRY.

151. Proposed by FRANK A. GIFFIN, Assistant in Mathematics, University of Colorado, Boulder, Col.

A point at a distance of 1 inch, 2 inches, and $2\frac{1}{2}$ inches, respectively, from three corners of a square. Construct the square. Also solve for the general distances a, b, c.

II. Solution by MARCUS BAKER, U. S. Coast Survey, Washington, D. C.

In the June-July number of the MONTHLY was published an *analysis* of this *without construction* as called for. The following is presented as an analysis *and* construction.

This is a variation of Rutherford's problem, which is: Given the distances a, b, c, of any point in the plane of an equilateral triangle, from the vertices of that triangle required to construct it. In the problem here proposed a right angled isosceles triangle replaces the equilateral triangle of Rutherford's problem.

Analysis. The triangle ABC is half of a square whose side is x, *i. e.* it is an isosceles triangle right angled at A. Let P be any point. Join P to A, B, and C by the lines a, b, and c, dividing the original triangle into three triangles. Now imagine each of these three triangles folded over its corresponding side of the original triangle in such wise that P falls at P_a, P_b, P_c. Complete the figure. Then there results a pentagon whose area is obviously twice that of ABC, *i. e.* equals x^2. This pentagon is composed of three triangles, viz:

P_cP_aB,	sides b, b, $b\sqrt{2}$,	area$=\frac{1}{2}b^2$
P_aP_bC,	sides c, c, $c\sqrt{2}$,	area$=\frac{1}{2}c^2$
$P_aP_bP_c$,	sides $b\sqrt{2}$, $c\sqrt{2}$, $2a$,	area$=$

$$\left[\left(\frac{b}{\sqrt{2}}+\frac{c}{\sqrt{2}}+a\right)\left(-\frac{b}{\sqrt{2}}+\frac{c}{\sqrt{2}}+a\right)\left(\frac{b}{\sqrt{2}}-\frac{c}{\sqrt{2}}+a\right)\left(\frac{b}{\sqrt{2}}+\frac{c}{\sqrt{2}}-a\right)\right]^{\frac{1}{2}}$$

Construction. From the above analysis it is obvious that we must first

construct the triangle $P_aP_bP_c$, whose sides are known, and then upon $b\surd 2$ and $c\surd 2$ construct isosceles right triangles. The vertices of these triangles (at the right angles) and the middle of the side $2c$ are the vertices of the required triangle.

Number of Solutions. In constructing $P_aP_bP_c$ we may take for its sides

	$a\surd 2$,	$b\surd 2$,	$2c$ ....(1).
or	$a\surd 2$,	$2b$,	$c\surd 2$ ...(2).
or	$2a$	$b\surd 2$,	$c\surd 2$....(3).

In the special problem before us $a=1$, $b=2$, $c=2\frac{1}{2}$; whence

1.414,	2.828,	5. ....(1).
1.414,	4. ,	3.535....(2).
2, ,	2.828,	3,535....(3).

In the first case there is no real solution.

In the second case P falls *without* the triangle, and $x=2\frac{7}{16}$.

In the third case P falls *within* the triangle, and $x=2\frac{13}{16}$.

The values are derived by scaling off from the figures. The construction here given is for the third case, the unit being one centimeter.

154. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Miss.

The angle between the edge of a trihedral angle and the bisector of the opposite face angle is less than, equal to, or greater than half the sum of the other two face angles, according as it is itself acute, right, or obtuse.

Solution by the PROPOSER.

Let $S-ABC$ be a trihedral angle and SD the bisector of the face angle ASB.

CASE I. $\angle DSC <$ a right angle. (See Fig. 1.)

To prove $\angle DSC < \frac{1}{2}(\angle ASC + \angle BSC)$.

From C, any point of the edge SC, draw CD perpendicular to SD.

Through D, in the face ASB, draw AB perpendicular to SD. Then, SD is perpendicular to the plane ABC.

Connect S with F and E.

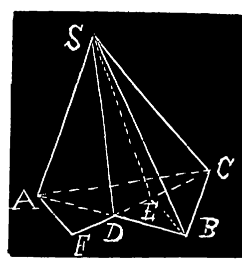

Fig. 1.

Comparing the right triangles AFD and BED, $AD=BD$ (since triangle ASD=triangle BSD), and the vertical angles at D are equal. Hence, the triangles are equal, and $DE=DF$.

It follows that right triangles FSD and ESD are equal, and $\angle FSD = \angle ESD$.

Now, since SD is perpendicular to the plane ABC, the plane DSC is perpendicular to the plane ABC. Therefore BE and AF, which lie in one of these planes and are perpendicular to their intersection, are perpendicular to the other plane, DSC.